

Non-singular Cyclic Cosmology without Phantom Menace

Yi-Fu Cai¹ and Emmanuel N. Saridakis²

¹ *Department of Physics, Arizona State University, Tempe, AZ 85287, USA*

email: ycai21@asu.edu

² *Center for Astrophysics, Space Physics and Engineering Research,
Physics Department, Baylor University, Waco, TX 76798-7310, USA*

email: Emmanuel.Saridakis@baylor.edu

Abstract

In this article we review recent developments of cyclic cosmology. A typical non-singular cyclic model within General Relativity requires a non-conventional fluid with negative effective energy density, in order to cancel the matter component and lead to a non-singular bounce. However, the existence of such a non-conventional fluid usually leads to quantum instabilities and makes the theory ill-defined. In the present work we follow the alternative way, obtaining two scenarios of non-singular cyclic cosmological evolutions in the context of gravitational theories beyond General Relativity. The degrees-of-freedom examination reveals that these two models are free of the Phantom Menace. Our analysis illustrates that, if cyclic cosmology describes overall universe, a theory of gravity beyond Einstein may be expected.

Keywords: non-singular cosmology, modified gravity, bounce, Horava-Lifshitz gravity

15 pages, 2 figures, 0 tables

Non-singular Cyclic Cosmology without Phantom Menace

Emmanuel N. Saridakis. email: Emmanuel.Saridakis@baylor.edu

Non-singular Cyclic Cosmology without Phantom Menace

I. INTRODUCTION

Big Bang cosmology, considered as the “standard model of the universe”, has achieved numerous successes in explaining the overall universe. However, a crucial unexplainable issue of this paradigm is the existence of the Big Bang itself, which suggests that our universe was born from a spacetime singularity. At that moment, physical quantities like density, pressure, temperature, spacetime curvature etc, were infinite and thus theoretically ill-defined. Additionally, open geometries should be totally excluded, since they correspond to infinite proper three-volume at the singularity. As a consequence, there has been a lot of effort in resolving this problem through quantum gravity effects or effective field theory techniques.

A phenomenological solution to the cosmic singularity problem may be provided by non-singular bouncing cosmologies (Tolman 1931). Such scenarios have been constructed through various approaches to modified gravity (Mukhanov:1991zn), such as the Pre-Big-Bang (Veneziano 1991, Gasperini & Veneziano 1993) and the Ekpyrotic (Khoury et al. 2001, Khoury et al. 2002) models, gravity with higher order corrections (Brustein & Madden 1998, Biswas et al. 2006), braneworld scenarios (Shtanov & Sahni 2003, Saridakis 2009), non-relativistic gravity (Cai & Saridakis 2009, Saridakis 2010), $f(T)$ modified gravity (Chen et al. 2011, Cai et al. 2011b) and loop quantum cosmology (Bojowald 2001). Non-singular bounces may be alternatively investigated using effective field description in the General Relativity background, introducing matter fields violating the null energy condition, the so-called Phantom fields, leading to observable predictions too (Cai et al. 2007, Cai et al. 2008), introducing non-conventional mixing terms (Saridakis & Ward 2009, Saridakis & Sushkov 2010, Saridakis & Weller 2010), or in the frame of a close universe (Martin & Peter 2003). However, these General-Relativity bounce-models suffer from the severe problem of quantum instability due to the Phantom field (Carroll et al. 2003, Cline et al. 2004), since once such a field with a negative kinetic term is introduced the vacuum becomes unstable (note also that the very concept of a negative kinetic energy itself sounds unphysical). In summary, the extension to gravitational theories beyond General Relativity seems to be the plausible and safest solution to the singularity problem.

An interesting extension of bouncing scenarios is the (old) paradigm of cyclic cosmology, in which the universe experiences the periodic sequence of contractions and expansions (Tolman 1934). Cyclic cosmology has gained new interest after the appearance of Quasi Steady State cosmology

(Hoyle et al. 1993, Hoyle et al. 1994), in which the C-field (Creation-field) is responsible for the repeated cycles. Furthermore, it has been revived in the past few years (Steinhardt & Turok 2002a, Steinhardt & Turok 2002b), since it brings different insights for the origin of the observable universe (see Novello & Bergliaffa 2008 for a review).

Let us first analyze the general features of a cosmological bounce and turnaround. The basic picture for the evolution of a cyclic universe can be shown below:

$$\dots \text{bounce} \xrightarrow{\text{expanding}} \text{turnaround} \xrightarrow{\text{contracting}} \text{bounce} \dots$$

Whether a universe is expanding or contracting depends on the positivity of the Hubble parameter ($H \equiv \dot{a}/a$ with a the scale factor of the universe). In the contracting phase that exists prior to the bounce, the Hubble parameter H is negative, while in the expanding one that exists after it we have $H > 0$. By making use of the continuity equations it follows that at the bounce point $H = 0$. Finally, it is easy to see that throughout this transition $\dot{H} > 0$. On the other hand, for the transition from expansion to contraction, that is for the cosmological turnaround, we have $H > 0$ before and $H < 0$ after, while exactly at the turnaround point we have $H = 0$. Throughout this transition $\dot{H} < 0$.

An oscillating universe realized in spatially flat geometry described by Einstein gravity has been studied in (Xiong et al. 2008, Cai et al. 2010), by making use of a quintom matter (Feng et al. 2005), which involves a ghost degree of freedom, and thus it cancels the contribution of conventional matter fields. Unfortunately, these types of models suffer from the problem of quantum instability due to an unbounded vacuum state from below, which is the so called ‘‘Phantom Menace’’ (Carroll et al. 2003, Cline et al. 2004).

Therefore, it is better to follow the alternative recipe to obtain a bounce, a turnaround and in general cyclic behavior, namely to acquire extra terms in the gravitational side of the Friedmann equations for H^2 and \dot{H} , in order for the aforementioned requirements to be fulfilled. Thus, we need to construct suitable generalizations of General Relativity, that are capable of inducing such novel terms. In the present work we will describe two possible scenarios to obtain cyclic behavior. Particularly, these two models are free of the Phantom Menace and the perturbations are able to pass through the bouncing point smoothly and without pathology.

II. PHANTOM MENACE OF CYCLIC COSMOLOGY IN GENERAL RELATIVITY

Before proceeding to our modified-gravity cyclic scenarios, in this section we briefly show why the cyclicity realization in the context of General Relativity is problematic. The simplest scenario of a cyclic universe in the frame of Einstein gravity is constructed in terms of the double-field quintom model (Xiong et al. 2008). The first field is a canonical one, with a positive kinetic term, called Quintessence, while the second possesses a negative kinetic term and it is called Phantom.¹ Its dynamics can be described by an action of the form

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \partial_\mu \psi \partial^\mu \psi - V(\phi, \psi) \right], \quad (1)$$

where ϕ and ψ are respectively the canonical and phantom fields, and the flat Friedmann-Robertson-Walker (FRW) metric reads as $ds^2 = dt^2 - a^2(t) dx^i dx^i$. In the framework of FRW cosmology we can obtain the energy density and pressure of the scenario as

$$\rho = \frac{1}{2}(\dot{\phi}^2 - \dot{\psi}^2) + V(\phi, \psi), \quad p = \frac{1}{2}(\dot{\phi}^2 - \dot{\psi}^2) - V(\phi, \psi). \quad (2)$$

These two quantities determine the evolution of the universe through the Friedmann equations

$$H^2 = \frac{\rho}{3M_p^2}, \quad \dot{H} = -\frac{\rho + p}{2M_p^2}, \quad (3)$$

where M_p is the reduced Planck mass (in the following we use the convention $M_p = 1/\sqrt{8\pi G}$, $c = 1$ and $\hbar = 1$). Finally, the two scalars satisfy the Klein-Gordon equations.

Phenomenologically, a general form of the potential for a renormalizable model includes operators with dimension 4 or less, consisting of various powers of the scalar fields. We impose a Z_2 symmetry, that is the potential remains invariant under the simultaneous transformations $\phi \rightarrow -\phi$ and $\psi \rightarrow -\psi$. For a detailed quantitative study we assume the potential form

$$V(\phi, \psi) = (\Lambda_0 + \lambda\phi\psi)^2 + \frac{1}{2}m^2\phi^2 - \frac{1}{2}m^2\psi^2, \quad (4)$$

where λ is a dimensionless constant describing the interaction between the two fields, and Λ_0 a constant with dimension of $[\text{mass}]^2$.

In our scenario the two scalars dominate the universe alternately. In order to present this behavior more transparently, we assume, without loss of generality, that the universe starts an expansion from a bounce point. It is then phantom-dominated and its energy density increases.

¹ This is equivalent to the Lee-Wick model in particle physics, which involves higher derivative terms and thus can yield a bouncing solution when applied into cosmology (Cai et al. 2009a).

However, a quintessence-dominated stage follows, and the energy density reaches a maximal value, after which it decreases. When it arrives at zero, the turnaround is realized and the universe enters into a contracting phase. Therefore, from the bounce to the turnaround the universe needs to transit from the phantom-dominated phase into the quintessence-dominated one, or vice versa. In order to describe the whole evolution explicitly, in Fig. 1 we provide the evolutions for the energy density and the scale factor of such a cyclic universe. We observe that during each cycle the universe is dominated by Quintessence and Phantom alternately.

[Insert Figure 1 Here]

However, this model suffers from a severe problem of quantum instability due to the Phantom field (Carroll et al. 2003, Cline et al. 2004). It is well known that a quantum field theory can be well defined based on the existence of a stable vacuum. In such a vacuum, pairs of virtual particles can be produced through quantum fluctuations and then annihilate rapidly. This implies that, for all fields, the kinetic terms ought to be positively defined, so that the virtual particles will not become real ones and the stability of the vacuum will be preserved. Thus, once a Phantom field with a negative kinetic term is introduced, the vacuum becomes unstable. Even assuming that ghosts interact gravitationally, the vacuum is able to produce a pair of virtual gravitons and then decay into a pair of ghosts and a pair of other particles such as photons. In this case, one can observe that gravitational radiation can be emitted from “nothing” and its density grows exponentially. In summary, it seems that cyclicity in the context of Einstein gravity, and thus under the necessary introduction of Phantom fields, suffers inherently from ghost instabilities². The plausible way out is to extend to gravitational theories beyond General Relativity.

III. CYCLICITY IN A MODEL OF NON-RELATIVISTIC QUANTUM GRAVITY

Motivated by a recent work (Horava 2009), one realizes that a scenario of power-counting renormalizable gravity may be achieved just by adding higher-order spatial derivative terms. The original model, which is the so-called Hořava-Lifshitz gravity, suffers from problems such as the

² We mention here that in the context of General Relativity one may alternatively remove the Big Bang singularity and enable a cyclic cosmology by introducing a nonlinear electro-dynamical Lagrangian (De Lorenci et al. 2002) in the right-hand-side of field equations. This scenario was analyzed in astrophysics in detail (see e.g. (Mosquera Cuesta & Salim 2004a, Mosquera Cuesta & Salim 2004b)), and its application to black hole physics was studied in (Corda & Mosquera Cuesta 2010).

over-constraining in UV region, being not compatible with current observations even in the IR limit. Additionally, painstaking latest analysis of cosmic Gamma Ray Bursts impose very stringent constraints on the Lorentz violation (Laurent et al. 2011), which is a key element of Hořava-Lifshitz gravity.

The logic of an effective field theory suggests that a complete action of gravity could include all possible terms consistent with the imposed symmetries, and the dimensions of these terms ought to be bounded due to renormalization. In the frame of 4-dimensional spacetime, a renormalizable term may allow for 6th-order spatial derivatives at most (Horava 2009). As a sacrifice, the Lorentz symmetry has to be abandoned, but it may appear as an emergent one at low-energy scales.

From these we deduce that one could use a modified action which involves all the permitted terms (Cai & Saridakis 2009, Saridakis 2010):

$$\Delta S_g = \frac{1}{16\pi G} \int dt d^3x \sqrt{g} N \left(\alpha_1 \bar{R}_{ij} \bar{R}^{ij} + \alpha_2 \bar{R}^2 + \alpha_3 \nabla_i \bar{R}_{jk} \nabla^i \bar{R}^{jk} + \alpha_4 \nabla_i \bar{R}_{jk} \nabla^j \bar{R}^{ki} + \alpha_5 \nabla_i \bar{R} \nabla^i \bar{R} \right), \quad (5)$$

where K_{ij} is the extrinsic curvature and \bar{R} is the three-dimensional Ricci scalar. The dynamical variables are the lapse and shift functions, N and N_i respectively, and the spatial metric g_{ij} (roman letters indicate spatial indices) writes as $ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$. Focusing on cosmological frameworks with an FRW metric we use $N = 1$, $g_{ij} = a^2(t)\gamma_{ij}$, $N^i = 0$, with $\gamma_{ij} dx^i dx^j = \frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2$, where $k = -1, 0, 1$ corresponds to open, flat, and closed geometry respectively. Finally, we can insert a matter component in the scenario, namely a canonical scalar ϕ with action

$$S_m = \int dt d^3x \sqrt{g} N \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] \quad (6)$$

into action (5), with $V(\phi)$ its potential, $\rho_m \equiv \frac{\dot{\phi}^2}{2} + V(\phi)$ its energy density and $p_m \equiv \frac{\dot{\phi}^2}{2} - V(\phi)$ its pressure. We stress that the above construction is free of Phantom fields.

By varying N and g_{ij} we obtain the Friedmann equations

$$H^2 = \frac{8\pi G}{3} \left[\rho_m + \rho_k + \rho_{dr} \right], \quad (7)$$

$$\dot{H} + \frac{3}{2} H^2 = -4\pi G \left[p_m - \frac{1}{3} \rho_k + \frac{1}{3} \rho_{dr} \right], \quad (8)$$

which are effectively the same as (3), but with the extra contributions of the curvature term $\rho_k \equiv -\frac{3k}{8\pi G a^2}$, and of the term:

$$\rho_{dr} \equiv -\frac{3(\alpha_1 + 3\alpha_2)k^2}{4\pi G a^4}. \quad (9)$$

This term is the so called “dark radiation”, since it possesses effective equation-of-state parameter $w = 1/3$, that is the same as normal radiation, but it exhibits a negative energy density when $\alpha_1 + 3\alpha_2 > 0$. Note that it exists only when the spatial geometry is not flat (Brandenberger 2009b). In summary, in the scenario at hand we obtain a component of negative energy density, and thus able to drive cyclic behavior, of pure gravitational origin, without the use of any Phantom field and therefore free of the ghost instability problem.

Observing the Friedmann equations, and having in mind the general requirements for the realization of cyclic behavior described in the Introduction, it is easy to see that these can be easily fulfilled in the model at hand. In particular, we have two choices. The first is to consider a specific matter potential $V(\phi)$, and then try to determine the parameter space that allows for the fulfillment of the requirements. However, it would be better to suitably determine (reconstruct) $V(\phi)$ in order to acquire cyclicity independently of the parameter values. Therefore, imposing a known scale factor $a(t)$ possessing an oscillatory behavior, both $H(t)$ and $\dot{H}(t)$ are straightforwardly known, and thus we can use the Friedmann equations together with the pressure and energy density of the scalar field, in order to extract the relations for $\phi(t)$. Finally, eliminating time between ϕ and V we extract the explicit profile of the potential $V(\phi)$. $V(\phi)$ will have an oscillating form too, since a non-oscillatory $V(\phi)$ would be physically impossible to generate an infinitely oscillating scale factor and a universe with a time-symmetry.

The aforementioned bottom to top approach was enlightening about the form of the scalar potential that leads to a cyclic cosmological behavior. Alternatively, we can perform the above procedure the other way around, starting from a specific oscillatory $V(\phi)$ and resulting to an oscillatory $a(t)$. As a specific example we consider the simple case

$$V(\phi) = V_0 \sin(\omega_V \phi) + V_c. \quad (10)$$

In this case, the Friedmann equation (8) gives a differential equation for the scale factor which can be easily solved numerically. In Fig. 2 we depict the corresponding solution for $a(t)$ (and thus for $H(t)$) with $V_0 = 5.25$, $\omega_V = 0.25$ and $V_c = 5.25$, with $k = 1$, $\alpha_1 = 1$ and $\alpha_2 = 1$ (in units of M_p). The potential parameters have been chosen in order to acquire a cyclic universe with $a \approx 1$ at the bounce.

[Insert Figure 2 Here].

Note that the above specific example is just a simple representative of cyclic behavior in our

gravitational and cosmological construction, and it corresponds only to a sub-class of the whole set of cyclic models. Obviously, one can straightforwardly generalize the aforementioned procedure in any periodic model. Moreover, since the bounce solutions arise owing to the presence of a dark radiation component with negative energy density, they can also be obtained if ordinary radiation with positive energy density is present. When ordinary radiation is involved, it has to be generated from the reheating process of a primordial field such as the inflaton, or straightaway from the matter field ϕ . Therefore, its domination takes place only after reheating, of which the energy scale is much lower than the bounce scale, and thus the bounce remains unaffected. Finally, in the late-time evolution, normal radiation would be erased during matter-dominated period, and hence it will not affect the bounce solution in the next cycle.

We close this section by stressing that the scenario at hand is free of the ghost problem, since the higher-order derivatives are only introduced along spatial directions but not in the time-like ones. Furthermore, it does not suffer from the strong coupling problem which exists in Hořava-Lifshitz gravity, since the kinetic term for gravity is exactly the same as that of Einstein-Hilbert action. As a consequence, we conclude that there are no extra degrees of freedom. These features are significant advantages, necessary for a consistent phenomenological description of the overall universe.

IV. CYCLICITY IN MODIFIED GRAVITY WITH LAGRANGE MULTIPLIERS

Let us now present cyclicity in a different modified gravity scenario, namely under Lagrange multipliers (Mukhanov & Brandenberger 1992, Lim et al. 2010, Capozziello et al. 2010). For simplicity we focus on the flat FRW geometry (Cai & Saridakis 2011), although we could straightforwardly generalize our results to the non-flat case too.

We start with a conventional $f(R)$ -gravity, and we add a scalar field λ which is a Lagrange multiplier. In particular, the action reads:

$$S = \int d^4x \sqrt{-g} \left\{ f_1(R) - \lambda \left[\frac{1}{2} \partial_\mu R \partial^\mu R + f_2(R) \right] \right\}, \quad (11)$$

where $f_1(R)$ and $f_2(R)$ are two independent functions of the Ricci scalar R . Note that in the above action we have not included the matter content of the universe for simplicity, since this would significantly modify the multiplying terms of λ , making the subsequent reconstruction procedure technically very complicated.

Due to the Lagrange multiplier form, variation over λ leads to an important constraint, namely

$f_2(R) = \frac{1}{2}\dot{R}^2$. For $f_2(R) > 0$, this constraint can be solved as $t = \int^R \frac{dR}{\sqrt{2f_2(R)}}$ and inverting this relation with respect to R one can obtain the explicit $R(t)$. Thus, using also the definition of the Ricci scalar one obtains a differential equation in terms of $H(t)$, namely

$$6\dot{H}(t) + 12[H(t)]^2 = R(t), \quad (12)$$

the solution of which determines completely the cosmological behavior.

In order to provide a simple realization of cyclicity in this scenario, we start by imposing a desirable form of $H(t)$ that corresponds to a cyclic behavior. In particular, having described the general requirements for cyclicity in the Introduction, we choose $H(t)$ to be straightaway a sinusoidal function: $H(t) = A_H \sin(\omega_H t)$, which gives rise to a non-singular and oscillating scale factor of the form of

$$a(t) = A_{H0} \exp \left[-\frac{A_H \cos(\omega_H t)}{\omega_H} \right]. \quad (13)$$

Thus, inserting this relation into (12) we can easily reconstruct the form of $f_2(R)$ as

$$f_2(R) = \frac{1}{4}\omega_H^2 \left(48A_H^2 - 4R + 3\omega_H^2 \right) \left[2R - 3\omega_H^2 - \omega_H \sqrt{3(48A_H^2 - 4R + 3\omega_H^2)} \right]. \quad (14)$$

In summary, such an ansatz for $f_2(R)$ produces the cyclic universe with scale factor (13). Note that $f_2(R)$ has a remarkably simple form, and this is an advantage of the scenario at hand, since in conventional $f(R)$ -gravity one needs very refined and complicated forms of $f(R)$ in order to reconstruct a given cosmological evolution. However, the absence of matter evolution in a cyclic scenario is a disadvantage, since we cannot reproduce the epoch evolution of the universe. Therefore it would be necessary to extend the above formalism under matter inclusion, similarly to the case of non-relativistic cyclic cosmology, a procedure which proves quite complicated (Cai & Saridakis 2011). We mention here that a cyclic model of modified gravity involving Lagrange Multiplier was also studied in the context of nonlinear electro-dynamical system in (Corda 2008), and its inflationary solution was addressed in (Corda & Mosquera Cuesta 2011) (see also Starobinsky 1979).

We close this section referring to the absence of quantum instabilities in the scenario at hand. In principle, one may worry since the action (11) involves higher derivative terms, which could imply such instabilities. However, this is not the case since these annoying terms can be frozen by the perturbed constraint equation. Additionally, the vanishing of the (i, j) component of the perturbed Einstein equation allows to eliminate one degree of freedom, and therefore there is still only one mode of metric perturbation which is able to propagate freely. In particular, we assume

that a bouncing phase is realized slowly, and then the universe approaches a static phase around the bounce asymptotically. Under this assumption we find that the kinetic term of the perturbation is positively defined and canonical, and that the sound speed of the perturbation is unity, as it can be read from the coefficient before the gradient term (Cai & Saridakis 2011). Furthermore, in such a case we explicitly confirm that there exist only a single degree of freedom in Lagrange-multiplier modified $f(R)$ cosmology. In conclusion, the scenario at hand is free of instabilities, and thus it could be a candidate for the description of the overall universe.

V. FLUCTUATIONS THROUGH THE BOUNCE

A scenario of non-relativistic gravity is usually able to recover Einstein's General Relativity as an emergent theory at low-energy scales. Therefore, the cosmological fluctuations generated in this model should be consistent with those obtained in standard perturbation theory in the IR limit (Brandenberger 2009b). This result has been intensively discussed in the literature (see e.g. Cai et al. 2009a). In particular, the perturbation spectrum presents a scale-invariant profile if the universe has undergone a matter-dominated contracting phase (Cai & Zhang 2009a, Cai et al. 2009b, Cai et al. 2009c, Cai et al. 2011). However, the corrections in the modified action of the present work could lead to a modification of the dispersion relations of perturbations. This issue has been addressed in (Cai & Zhang 2009b), which shows that the spectrum in the UV regime may have a red tilt in a bouncing universe. Moreover, the perturbation modes cannot even enter the UV regime in the scenario of matter-bounce. Thus, the analysis of the cosmological perturbations in the IR regime is quite reliable. Finally, we mention that the gravitational wave astronomy may also give a hope to the potential test of modified gravity models (see Corda 2009).

Things become complicated but more interesting in a cyclic scenario. Usually, a particular perturbation mode in the contracting phase is dominated by its growing tendency, but in the expanding stage it becomes nearly constant on super-Hubble scales. Therefore, the metric perturbation is amplified on super-Hubble scales cycle by cycle (Piao 2009, Zhang et al. 2010), and moreover the slope of its spectral index is varying (Brandenberger 2009a). However, it is known that the contribution of fluctuations has to be much less than the background energy. This prohibits the metric perturbations to enter the next cycle if $\delta\rho/\rho \sim O(1)$, unless the universe can be separated into many parts independent of one another, each of which corresponding to a new universe and evolving up to next cycle, then separate again and so on. In this case, the model of cyclic universe may be viewed as a realization of the multiverse scenario (Erickson et al. 2007,

Lehners & Steinhardt 2009, Piao 2009, Zhang et al. 2010).

VI. CONCLUSIONS

In this article, we have reviewed generic features of non-singular cyclic cosmology. There exist two possible approaches to realize cyclicity. The first is to introduce non-conventional matter fields, such as quintom, however such a scenario typically suffers from the problem of quantum instability. In order to avoid this undesirable pathology, a much more promising recipe along the direction of extended Einstein gravity is expected. Thus, we constructed two explicit examples of realizing cyclic behavior without the Phantom menace. However, we should mention here that the whole discussion lies on the grounds of the cosmological principle, that is on the assumption that the matter distribution of the cosmos is homogeneous, which may not be the case according to latest analysis of Sloan Digital Sky Survey (Thomas et al. 2011). This point deserves further investigation.

The first scenario is based on the introduction of higher-order derivatives of curvature terms in spatial coordinates, but preserving the kinetic term of Einstein-Hilbert action unchanged. This model can yield an oscillatory scale factor of the universe without the ghost-presence and strong-coupling problems.

The second scenario is the $f(R)$ -gravity including a Lagrange multiplier field. Under the assumption of slow bounce, we find only one canonical degree of freedom of which the sound speed is unity. Thus, it is possible for the cosmological perturbations to evolve through the bounces without quantum instability, too.

In conclusion, the analysis of the present work indicates that extending to gravitational theories beyond General Relativity provides a simple and consistent way for the realization of cyclic cosmology. However, we must mention here that observationally, General Relativity is still the most compatible theory, both at the cosmological and Solar system scales, and that any distinguishable feature of modified gravity is very difficult to be verified. The motivation of the above investigation arises from the conceptual level, and it is just the singularity avoidance, which is a disadvantage of General Relativity.

Acknowledgments

Y.F.C. is supported by funds from the Department of Physics at Arizona State University. Y.F.C thanks McGill University for the hospitality when this work was initiated.

References

- Biswas, T., Mazumdar, A., Siegel, W. (2006). Bouncing universes in string-inspired gravity. *JCAP* **0603**, 009.
- Bojowald, M. (2001). Absence of singularity in loop quantum cosmology. *Phys. Rev. Lett.* **86**, 5227.
- Brandenberger, R. H. (2009). Processing of Cosmological Perturbations in a Cyclic Cosmology. *Phys. Rev. D* **80**, 023535.
- Brandenberger, R. (2009). Matter Bounce in Horava-Lifshitz Cosmology. *Phys. Rev. D* **80**, 043516.
- Brustein, R., Madden, R. (1998). A model of graceful exit in string cosmology. *Phys. Rev. D* **57**, 712.
- Cai, Y. F., Saridakis, E. N. (2009). Non-singular cosmology in a model of non-relativistic gravity. *JCAP* **0910**, 020.
- Cai, Y. F., Saridakis, E. N. (2011). Cyclic cosmology from Lagrange-multiplier modified gravity. *Class. Quant. Grav.* **28**, 035010.
- Cai, Y. F., Zhang, X. (2009). Evolution of Metric Perturbations in Quintom Bounce model. *JCAP* **0906**, 003.
- Cai, Y. F., Zhang, X. (2009). Primordial perturbation with a modified dispersion relation. *Phys. Rev. D* **80**, 043520.
- Cai, Y. F., Brandenberger, R., Zhang, X. (2011). The Matter Bounce Curvaton Scenario. *JCAP* **1103**, 003.
- Cai, Y. F., Qiu, T. t., Brandenberger, R., Zhang, X. m. (2009). A Nonsingular Cosmology with a Scale-Invariant Spectrum of Cosmological Perturbations from Lee-Wick Theory. *Phys. Rev. D* **80**, 023511.
- Cai, Y. F., Saridakis, E. N., Setare, M. R., Xia, J. Q. (2010). Quintom Cosmology: Theoretical implications and observations. *Phys. Rept.* **493**, 1.
- Cai, Y. F., Xue, W., Brandenberger, R., Zhang, X. (2009). Non-Gaussianity in a Matter Bounce. *JCAP* **0905**, 011.

- Cai, Y. F., Xue, W., Brandenberger, R., Zhang, X. (2009). Thermal Fluctuations and Bouncing Cosmologies. *JCAP* **0906**, 037.
- Cai, Y. F., Chen, S. -H., Dent, J. B., Dutta, S., Saridakis, E. N. (2011). Matter Bounce Cosmology with the $f(T)$ Gravity. [arXiv:1104.4349 [astro-ph.CO]] (to appear in *Class. Quant. Grav.*)
- Cai, Y. F., Qiu, T., Brandenberger, R., Piao, Y. S., Zhang, X. (2008). On Perturbations of Quintom Bounce. *JCAP* **0803**, 013.
- Cai, Y. F., Qiu, T., Piao, Y. S., Li, M., Zhang, X. (2007). Bouncing Universe with Quintom Matter. *JHEP* **0710**, 071.
- Capozziello, S., Matsumoto, J., Nojiri, S., Odintsov, S. D. (2010). Dark energy from modified gravity with Lagrange multipliers. *Phys. Lett. B* **693**, 198.
- Carroll, S. M., Hoffman, M., Trodden, M. (2003). Can the dark energy equation-of-state parameter w be less than -1 ?. *Phys. Rev. D* **68**, 023509.
- Chen, S. -H., Dent, J. B., Dutta, S., Saridakis, E. N. (2011). Cosmological perturbations in $f(T)$ gravity. *Phys. Rev. D* **83**, 023508.
- Cline, J. M., Jeon, S., Moore, G. D. (2004). The phantom menaced: Constraints on low-energy effective ghosts. *Phys. Rev. D* **70**, 043543.
- Corda, C. (2008). An oscillating Universe from the linearized R^2 theory of gravity. *Gen. Rel. Grav.* **40**, 2201.
- Corda, C. (2009). Interferometric detection of gravitational waves: the definitive test for General Relativity. *Int. J. Mod. Phys. D* **18**, 2275.
- Corda, C., Mosquera Cuesta, H. J. (2010). Removing black-hole singularities with nonlinear electrodynamics. *Mod. Phys. Lett. A* **25**, 2423.
- Corda, C., Mosquera Cuesta, H. J. (2011). Inflation from R^2 gravity: a new approach using nonlinear electrodynamics. *Astropart. Phys.* **34**, 587.
- De Lorenci, V. A., Klippert, R., Novello, M., M. Salim, J. (2002). Nonlinear electrodynamics and FRW cosmology. *Phys. Rev. D* **65**, 063501.
- Erickson, J. K., Gratton, S., Steinhardt, P. J., Turok, N. (2007). Cosmic Perturbations Through the Cyclic Ages. *Phys. Rev. D* **75**, 123507.
- Feng, B., Wang, X. L., Zhang, X. M. (2005). Dark Energy Constraints from the Cosmic Age and Supernova. *Phys. Lett. B* **607**, 35.
- Gasperini, M., Veneziano, G. (1993) Pre - big bang in string cosmology. *Astropart. Phys.* **1**, 317.

- Horava, P. (2009). Quantum Gravity at a Lifshitz Point. *Phys. Rev. D* **79**, 084008.
- Hoyle, F., Burbidge, G., Narlikar, J. V. (1993). A quasi-steady state cosmological model with creation of matter. *Astrophys.J.* **410**, 437.
- Hoyle, F., Burbidge, G., Narlikar, J. V. (1994). Astrophysical deductions from the quasi-steady state cosmology. *Mon.Not.Roy.Astron.Soc.* **267**, 1007.
- Khoury, J., Ovrut, B. A., Steinhardt, P. J., Turok, N. (2001). The ekpyrotic universe: Colliding branes and the origin of the hot big bang. *Phys. Rev. D* **64**, 123522.
- Khoury, J., Ovrut, B. A., Seiberg, N., Steinhardt, P. J., Turok, N. (2002). From big crunch to big bang. *Phys. Rev. D* **65**, 086007.
- Laurent, P., Gotz, D., Binetruy, P., Covino, S., Fernandez-Soto, A. (2011). Constraints on Lorentz Invariance Violation using INTEGRAL/IBIS observations of GRB041219A. *Phys. Rev. D* **83**, 121301.
- Lehners, J. L., Steinhardt, P. J. (2009). Dark Energy and the Return of the Phoenix Universe. *Phys. Rev. D* **79**, 063503.
- Lim, E. A., Sawicki, I., Vikman, A. (2010). Dust of Dark Energy. *JCAP* **1005**, 012.
- Martin, J., Peter, P. (2003). Parametric amplification of metric fluctuations through a bouncing phase. *Phys. Rev. D* **68**, 103517.
- Mukhanov, V. F., Brandenberger, R. H. (1992). A Nonsingular universe. *Phys. Rev. Lett.* **68**, 1969.
- Mosquera Cuesta, H. J., Salim, J. M. (2004). Nonlinear electrodynamics and the surface redshift of pulsars. *Astrophys. J.* **608**, 925.
- Mosquera Cuesta, H. J., Salim, J. M. (2004). Nonlinear electrodynamics and the gravitational redshift of pulsars. *Mon. Not. Roy. Astron. Soc.* **354**, L55.
- Novello, M., Bergliaffa, S. E. P. (2008). Bouncing Cosmologies. *Phys. Rept.* **463**, 127.
- Piao, Y. S. (2009). Proliferation in Cycle. *Phys. Lett. B* **677**, 1.
- Saridakis, E. N. (2009). Cyclic Universes from General Collisionless Braneworld Models. *Nucl. Phys. B* **808**, 224.
- Saridakis, E. N. (2010). Horava-Lifshitz Dark Energy. *Eur. Phys. J. C* **67**, 229.
- Saridakis, E. N., Sushkov, S. V. (2010). Quintessence and phantom cosmology with non-minimal derivative coupling. *Phys. Rev. D* **81**, 083510.
- Saridakis, E. N., Ward, J. (2009). Quintessence and phantom dark energy from ghost D-branes. *Phys. Rev. D* **80**, 083003.
- Saridakis, E. N., Weller, J. M. (2010). A Quintom scenario with mixed kinetic terms.

- Phys. Rev. D **81**, 123523.
- Shtanov, Y., Sahni, V. (2003). Bouncing braneworlds. Phys. Lett. B **557**, 1.
- Steinhardt, P. J., Turok, N. (2002). Cosmic evolution in a cyclic universe. Phys. Rev. D **65**, 126003.
- Steinhardt, P. J., Turok, N. (2002). A cyclic model of the universe. Science **296**, 1436.
- Starobinsky, A. A. (1979). Relict Gravitation Radiation Spectrum and Initial State of the Universe. JETP Lett. **30** (1979) 682.
- Tolman, R. C. (1931). On the Problem of the Entropy of the Universe as a Whole. Phys. Rev. **37**, 1639.
- Tolman, R. C. (1934). Relativity, Thermodynamics and Cosmology. Oxford University Press, Oxford, UK.
- Thomas, S. A., Abdalla, F. B., Lahav, O. (2011). Excess Clustering on Large Scales in the MegaZ DR7 Photometric Redshift Survey. Phys. Rev. Lett. **106**, 241301.
- Veneziano, G. (1991). Scale Factor Duality For Classical And Quantum Strings. Phys. Lett. B **265**, 287.
- Xiong, H. H., Cai, Y. F., Qiu, T., Piao, Y. S., Zhang, X. (2008). Oscillating universe with quintom matter. Phys. Lett. B **666**, 212.
- Zhang, J., Liu, Z. G., Piao, Y. S. (2010). Amplification of curvature perturbations in cyclic cosmology. Phys. Rev. D **82**, 123505.

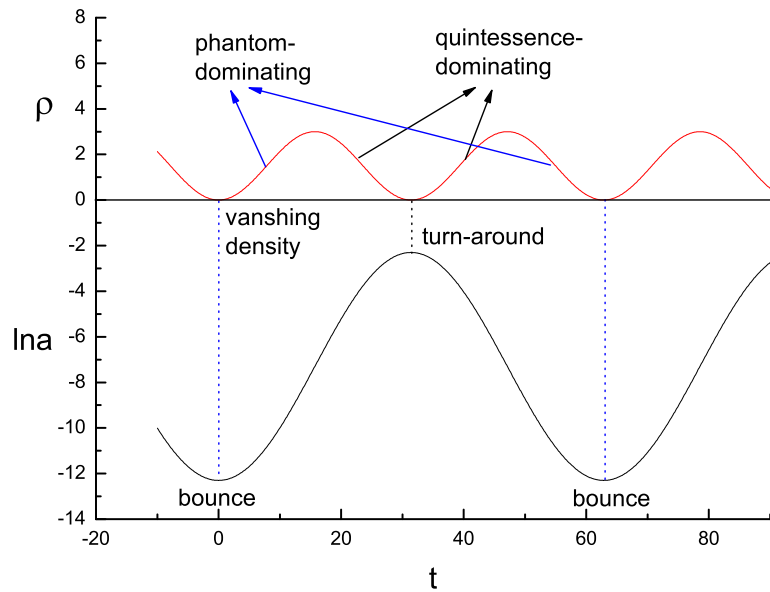


FIG. 1: *The evolution of a (symmetric) cyclic universe (Xiong et al. 2008). The scale factor of the universe oscillates between the minimal and maximal value. For each cycle the quintessence-like and phantom-like components dominate alternately.*

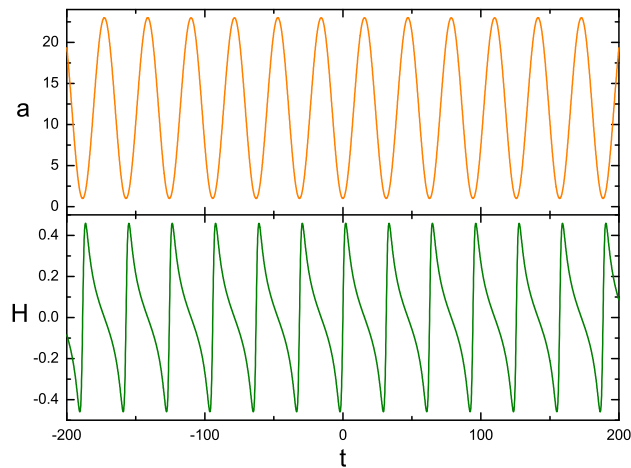


FIG. 2: *The evolution of the scale factor $a(t)$ and of the Hubble parameter $H(t)$, for a scalar potential of the ansatz (10) with $V_0 = 5.25$, $\omega_V = 0.25$ and $V_c = 5.25$, with $k = 1$, $\alpha_1 = 1$ and $\alpha_2 = 1$ (Cai & Saridakis 2009, Saridakis 2010).*